Designing Digital Communication Systems

by

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Introduction

This article is intended to serve as a "road map" for outlining typical steps to be considered in meeting the bandwidth, power, and error-performance requirements of a digital communication system. The criteria for choosing modulation and coding schemes, based on whether a system is bandwidth-limited or power-limited, are reviewed for several system examples, and the article emphasizes the subtle but straightforward relationships that exist when transforming from data bits to channel bits to symbols to chips.

The design of any digital communication system begins with a description of the channel (received power, available bandwidth, noise statistics and other impairments, such as fading), and a definition of the system requirements (data rate and error performance). Given the channel description, we need to determine design choices that best match the channel and meet the performance requirements. An orderly set of transformations and computations has evolved to aid in characterizing a system's performance. This article examines three system examples: a bandwidth-limited uncoded system, a power-limited uncoded system, and a bandwidth-limited and power-limited coded system. We deal with real-time communication systems, in which the term *coded* (or *uncoded*) refers to the presence (or absence) of error-correction coding schemes involving the use of *redundant bits* and expanded bandwidth.

Two primary communications resources are the *received power* and the *available transmission bandwidth*. In many communication systems, one of these resources may be more precious than the other, and hence most systems can be classified as either bandwidth-limited or power-limited. In bandwidth-limited systems, spectrally-efficient modulation techniques can be used to save bandwidth at the expense of power, whereas, in power-limited systems, power-efficient modulation techniques can be used to save power at the expense of bandwidth. In systems that are both bandwidth-limited and power-limited, error-correction coding (often called *channel coding*) can be used to save power or to improve error performance at the expense of bandwidth. Trellis-coded modulation (TCM) schemes can be used to improve the error performance of bandwidth-limited channels without *any* increase in bandwidth [1], but such schemes are not covered in this article.

The Bandwidth Efficiency Plane

Figure 1 shows a plot of bandwidth efficiency. The abscissa is the ratio of bit energy to noise-power spectral density, E_b/N_0, in decibels. The ordinate is the ratio of throughput, R in bit/s, that can be transmitted per hertz in a given bandwidth, W. The ratio R/W is called *bandwidth efficiency*, since its value reflects how efficiently the bandwidth resource is utilized. The plot in Figure 1 stems from the Shannon-Hartley Capacity Theorem [2-4], which can be stated as follows:

$$C = W \log_2\left(1 + \frac{S}{N}\right) \qquad (1)$$

where S/N is the ratio of received average signal power to noise power. When the logarithm is taken to the base 2, as shown, the capacity, C, is given in bit/s. The capacity of a channel defines the maximum number of bits that can be reliably sent per second over the channel. For the case where the data (information) rate, R, is equal to C, the curve separates a region of practical communication systems from a region where communication systems cannot operate reliably [3,4].

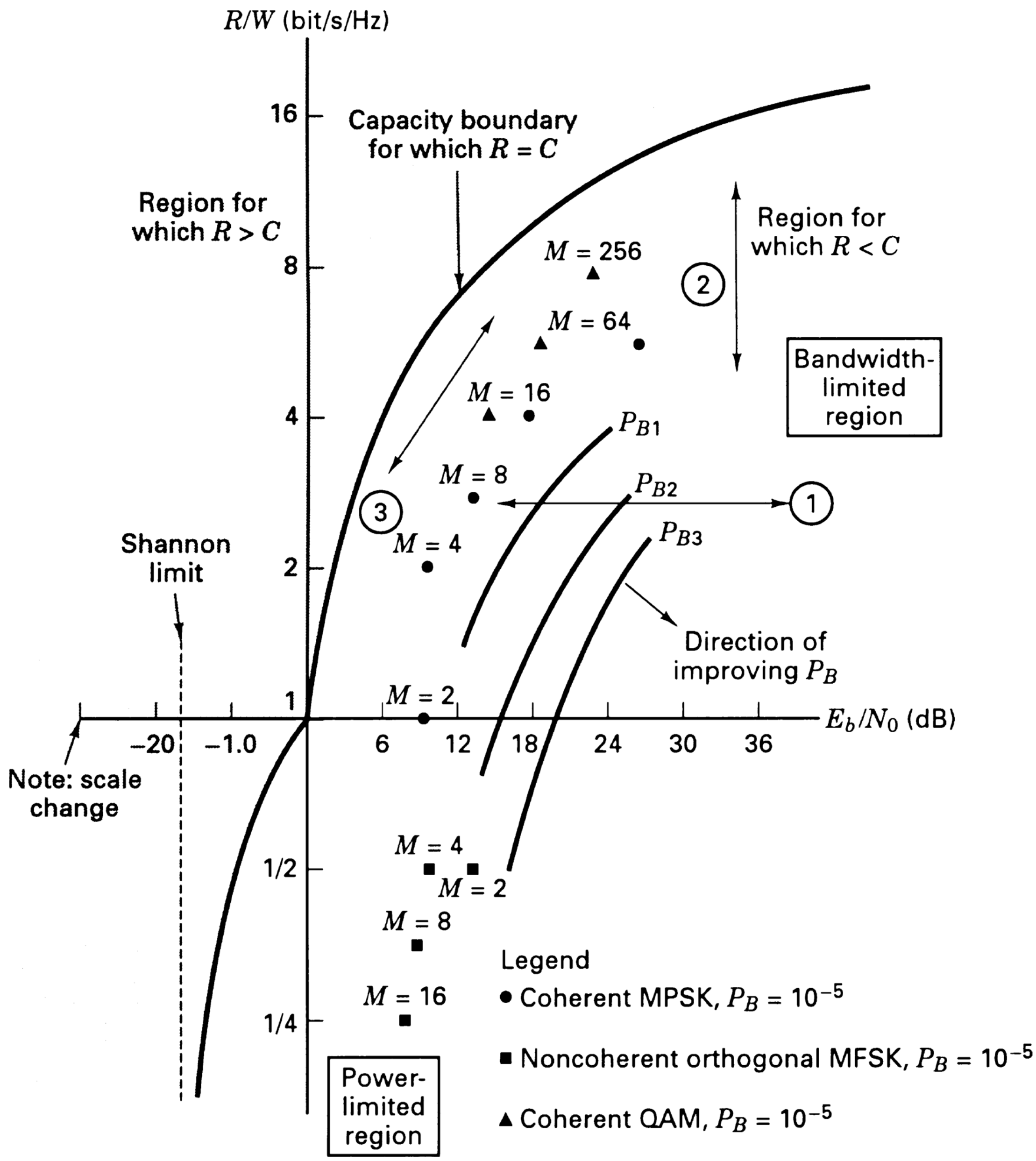

Figure 1

Bandwidth efficiency versus E_b/N_0.

M-ary Signaling

For signaling schemes that process k bits at a time, the signaling is called M-ary. Each symbol in an M-ary alphabet can be related to a unique sequence of k bits,

$$M = 2^k \quad \text{or} \quad k = \mathbf{log}_2 M \tag{2}$$

where M is the size of the alphabet. In the case of digital transmission, the term *symbol* refers to the member of the M-ary alphabet that is transmitted during each symbol duration, T_s. To transmit the symbol, it must be mapped onto an electrical voltage or current waveform. Because the transmission waveform represents the transmission symbol, the terms *symbol* and *waveform* are sometimes used interchangeably. Since one of M symbols or waveforms is transmitted during each symbol duration, T_s, the data rate, R in bit/s, can be expressed as follows:

$$R = \frac{k}{T_s} = \frac{\log_2 M}{T_s} \qquad \text{bit/s} \tag{3}$$

From Equation (3), we write that the *effective* time duration, T_b, of each bit in terms of the symbol duration, T_s, or the symbol rate, R_s, is

$$T_b = \frac{1}{R} = \frac{T_s}{k} = \frac{1}{k\,R_s} \tag{4}$$

Then, using Equations (3) and (4), we can express the symbol rate, R_s, in terms of the bit rate, R, as

$$R_s = \frac{R}{\log_2 M} \tag{5}$$

From Equations (3) and (4), it can be seen that any digital scheme that transmits $k = (\mathbf{log}_2 M)$ bits in T_s seconds, using a bandwidth of W Hz, operates at a bandwidth efficiency of

$$\frac{R}{W} = \frac{\log_2 M}{W\ T_s} = \frac{1}{W\ T_b} \qquad \text{bits/s/Hz} \tag{6}$$

where T_b is the effective time duration of each data bit.

Bandwidth-Limited Systems

From Equation (6), it can be seen that any digital communication system will become more bandwidth efficient as its WT_b product is decreased. Thus, signals with small WT_b products are often used with bandwidth-limited systems. For example, the Global System for Mobile (GSM) Communication uses Gaussian minimum shift keying (GMSK) modulation having a WT_b product equal to 0.3 Hz/bit/s [5], where W is the 3-dB bandwidth of a Gaussian filter.

For uncoded bandwidth-limited systems, the objective is to maximize the transmitted information rate within the allowable bandwidth, at the expense of E_b/N_0 (while maintaining a specified value of bit-error probability, P_B). On the bandwidth-efficiency plane of Figure 1 are plotted the operating points for coherent M-ary PSK (MPSK) at $P_B = 10^{-5}$. We will assume Nyquist (ideal rectangular) filtering at baseband [6], so that, for MPSK, the required double-sideband (DSB) bandwidth at an intermediate frequency (IF) is related to the symbol rate as follows

$$W = \frac{1}{T_s} = R_s \tag{7}$$

where T_s is the symbol duration and R_s is the symbol rate. The use of Nyquist filtering results in the *minimum* required transmission bandwidth that yields zero intersymbol interference; such ideal filtering gives rise to the name *Nyquist minimum bandwidth*. Note that the bandwidth of nonorthogonal signaling, such as MPSK or MQAM, does not depend on the density of the signaling points in the constellation—only on the speed of signaling. When a phasor is transmitted, the system cannot distinguish as to whether that signal arose from a sparse alphabet set or a dense alphabet set. It is this aspect of nonorthogonal signals that allows us to pack the signaling space densely and thus achieve improved bandwidth efficiency at the expense of power. From Equations (6) and (7), the bandwidth efficiency of MPSK modulated signals using Nyquist filtering can be expressed as follows:

$$\frac{R}{W} = \log_2 M \qquad \text{bits/s/Hz} \tag{8}$$

The MPSK points plotted in Figure 1 confirm the relationship shown in Equation (8). Note that MPSK modulation is a bandwidth-efficient scheme. As M increases in value, R/W also increases. From Figure 1, it can be verified that MPSK modulation can achieve improved bandwidth efficiency at the cost of increased E_b/N_0. Many highly bandwidth-efficient modulation schemes have been investigated [7], but such schemes are beyond the scope of this article.

Two regions, the bandwidth-limited region and the power-limited region, are shown on the bandwidth-efficiency plane of Figure 1. Notice that the desirable tradeoffs associated with each of these regions are not equitable. For the bandwidth-limited region, large R/W is desired; however, as E_b/N_0 is increased, the capacity boundary curve flattens out and ever-increasing amounts of additional E_b/N_0 are required to achieve improvement in R/W. A similar relationship is at work in the power-limited region. Here, a savings in E_b/N_0 is desired, but the capacity boundary curve is steep; to achieve a small reduction in required E_b/N_0 requires a large reduction in R/W.

Power-Limited Systems

For the case of power-limited systems in which power is scarce but system bandwidth is available (for example, a space communication link), the following tradeoffs are possible: (1) improved P_B at the expense of bandwidth for a fixed E_b/N_0; or (2) reduction in E_b/N_0 at the expense of bandwidth for a fixed P_B. A "natural" modulation choice for a power-limited system is M-ary FSK (MFSK). Plotted on Figure 1 are the operating points for noncoherent orthogonal MFSK modulation at $P_B = 10^{-5}$. For MFSK, the IF minimum bandwidth is given by

$$W = \frac{M}{T_s} = M\,R_s \tag{9}$$

where T_s is the symbol duration and R_s is the symbol rate. With MFSK, the required transmission bandwidth is expanded M-fold over binary FSK, since there are M different orthogonal waveforms, each requiring a bandwidth of $\mathbf{1}/T_S$. Thus, from Equations (6) and (9), the bandwidth efficiency of noncoherent MFSK signals can be expressed as follows:

$$\frac{R}{W} = \frac{\log_2 M}{M} \quad \text{bits/s/Hz} \tag{10}$$

The MFSK points plotted in Figure 1 confirm the relationship shown in Equation (10). Note that orthogonal signaling, such as MFSK modulation, is a bandwidth-expansive scheme. As M increases, R/W decreases. From Figure 1, it can be seen that MFSK modulation can be used for realizing a reduction in required E_b/N_0 at the cost of increased bandwidth.

It is important to emphasize that in Equations (7) and (8) for MPSK, and for all the MPSK points plotted in Figure 1, Nyquist (ideal rectangular) filtering has been assumed. Such filters are not realizable. For *realistic* channels and waveforms, the required transmission bandwidth must be *increased* in order to account for *realizable* filters. In each of the examples that follow, we consider radio channels, disturbed *only* by additive white Gaussian noise (AWGN), and having no other impairments. For simplicity, the modulation choice is limited to *constant-envelope types*—either MPSK or noncoherent orthogonal MFSK. Thus, for an *uncoded* system, if the channel is bandwidth-limited, MPSK is selected, and if the channel is power-limited, MFSK is selected. Note that, *when error-correction coding is considered,* modulation selection is more complex, because some coding techniques [8] can provide power/bandwidth tradeoffs more effectively than would be possible through the use of any *M*-ary modulation scheme.

Note, that in the most general sense, *M*-ary signaling can be regarded as a *waveform-coding* procedure. That is, whenever we select an *M*-ary modulation technique instead of a binary one, we *in effect* have replaced the binary waveforms with *better* waveforms – either better for bandwidth performance (MPSK), or better for power performance (MFSK). Even though orthogonal MFSK signaling can be thought of as being a coded system (it can be described as a first-order Reed-Muller code [9]), we shall here restrict our use of the term *coded system* to refer only to those traditional error-correction codes using redundancies, such as block codes or convolutional codes.

Requirements for MPSK and MFSK Signaling

The basic relationship between the symbol (or waveform) transmission rate, R_s, and the data rate, R, was shown in Equation (5) to be as follows:

$$R_s = \frac{R}{\log_2 M}$$

Using this relationship together with Equations (6) through (10), and a given data rate of R = 9600 bit/s, Table 1 has been compiled [4]. The table is a summary of symbol rate, minimum bandwidth, and bandwidth efficiency for MPSK and noncoherent orthogonal MFSK, for the values of M = 2, 4, 8, 16, and 32. Table 1 also includes the required values of E_b/N_0 to achieve a bit-error probability of 10^{-5} for MPSK and MFSK for each value of M shown. These E_b/N_0 entries were computed using relationships that are presented later in this article. The E_b/N_0 entries corroborate the tradeoffs shown in Figure 1. As M increases, MPSK signaling provides more bandwidth efficiency at the cost of increased E_b/N_0, while MFSK signaling allows for a reduction in E_b/N_0 at the cost of increased bandwidth. The next three sections are presented in the context of examples taken from Table 1.

Table 1

Symbol Rate, Minimum Bandwidth, Bandwidth Efficiency, and Required E_b/N_0 for MPSK and Noncoherent Orthogonal MFSK Signaling at 9600 bit/s

M	k	R (bit/s)	R_s (symb/s)	MPSK Minimum Bandwidth (Hz) (R/W=logM, W=R/logM)	MPS K R/W	MPSK E_b/N_0 (dB) $P_B = 10^{-5}$	Noncoherent Orthogonal MFSK Min Bandwidth (Hz) (R/W=logM/M, W=RM/logM)	MFS K R/W	MFSK E_b/N_0 (dB) $P_B = 10^{-5}$
2	1	9600	9600	9600	1	9.6	19,200	1/2	13.4
4	2	9600	4800	4800	2	9.6	19,200	1/2	10.6
8	3	9600	3200	3200	3	13.0	25,600	1/3	9.1
16	4	9600	2400	2400	4	17.5	38,400	1/4	8.1
32	5	9600	1920	1920	5	22.4	61,440	5/32	7.4

Example 1: Bandwidth-Limited Uncoded System

Suppose we are given a bandwidth-limited AWGN radio channel with an available bandwidth of $W = 4000$ Hz. Also, consider that the link constraints (transmitter power, antenna gains, path loss, and so on) result in the ratio of received signal power to noise-power spectral density, P_r/N_0, being equal to 53 dB-Hz. Let the required data rate, R, be equal to 9600 bit/s, and let the required bit-error performance, P_B, be *at most* 10^{-5}. The goal is to choose a modulation scheme that meets the required performance. In general, an error-correction coding scheme may be needed if none of the allowable modulation schemes can meet the requirements. However, in this example, we will see that the use of error-correction coding is unnecessary.

For any digital communication system, the relationship between received power to noise-power spectral density, P_r/N_0, and received bit-energy to noise-power spectral density, E_b/N_0, can be shown [4] to be the following:

$$\frac{P_r}{N_0} = \frac{E_b}{N_0} R \tag{11}$$

Solving for E_b/N_0 in decibels, we obtain this:

$$\frac{E_b}{N_0}\ (\text{dB}) = \frac{P_r}{N_0}\ (\text{dB-Hz})\ -R\ (\text{dB-bit/s}) \tag{12}$$

$$= 53 \text{ dB-Hz} - (10 \times \log_{10} 9600) \text{ dB-bit/s} = 13.2 \text{ dB (or 20.89)}$$

Since the required data rate of 9600 bit/s is much larger than the available bandwidth of 4000 Hz, the channel can be described as *bandwidth-limited.* We therefore select MPSK as our modulation scheme. Remember that we have confined the possible modulation choices to be constant-envelope types; without such a restriction, it would be possible to select a modulation type with greater bandwidth-efficiency. In an effort to conserve power, we next compute the *smallest possible* value of M such that the symbol rate is *at most* equal to the available bandwidth of 4000 Hz. From Table 1, it is clear that the smallest value of M meeting this requirement is $M = 8$. Our next task is to determine whether the required bit-error performance of $P_B \leq 10^{-5}$ can be met by using 8-PSK modulation alone, or whether it is necessary to also use an error-correction coding scheme. From Table 1, it can be seen that 8-PSK *alone* will meet the requirements, since the required E_b/N_0 listed for 8-PSK is less than the received E_b/N_0 that was calculated in Equation (12). However, imagine that we do not have Table 1. Let's see how to evaluate whether error-correction coding is necessary.

Figure 2 shows the basic modulator/demodulator (MODEM) block diagram summarizing the functional details of this design. At the modulator, the transformation from data bits to symbols yields an output symbol rate R_S, that is a factor ($\log_2 M$) smaller than the input data-bit rate R, as is seen in Equation (5).

Similarly, at the input to the demodulator, the symbol-energy to noise-power spectral density E_S/N_0 is a factor ($\log_2 M$) larger than E_b/N_0, since each symbol is made up of ($\log_2 M$) bits. Because E_S/N_0 is larger than E_b/N_0 by the same factor that R_s is smaller than R, we can expand Equation (11) as follows:

$$\frac{P_r}{N_0} = \frac{E_b}{N_0} R = \frac{E_s}{N_0} R_s \tag{13}$$

The demodulator receives a waveform (in this example, one of $M = 8$ possible phase shifts) during each time interval T_s. The probability that the demodulator makes a symbol error, $P_E(M)$, is well approximated by [10].

$$P_E(M) \approx 2Q\left[\sqrt{\frac{2E_s}{N_0}}\ \sin\left(\frac{\pi}{M}\right)\right] \quad \text{for } M > 2 \tag{14}$$

where Q(x), the complementary error function, is defined as [4] follows:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) du \tag{15}$$

A good approximation for $Q(x)$, valid for $x > 3$, is

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{16}$$

In Figure 2 and all the figures that follow, rather than show explicit probability relationships, the generalized notation $f(x)$ is used to indicate some functional dependence on x.

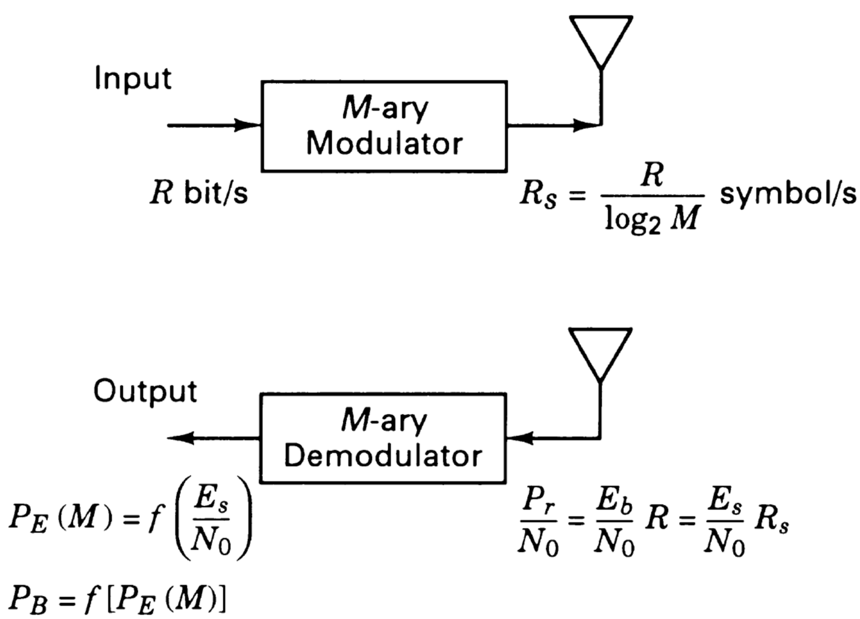

Figure 2

Basic modulator/demodulator (MODEM) without channel coding.

A traditional way of characterizing communication (power) efficiency or error performance in digital systems is in terms of the received E_b/N_0 in decibels. This E_b/N_0 description has become standard practice. However, recall that at the input to the demodulator/detector, there are no bits; there are only waveforms that have been assigned bit meanings. Thus, the received E_b/N_0 value represents a bit-apportionment of the arriving waveform energy. A more precise (but unwieldy) name would be the energy per *effective bit* versus N_0. To solve for $P_E(M)$ in Equation (14), we first need to compute the ratio of received symbol-energy to noise-power spectral density, E_S/N_0. Since from Equation (12), $E_b/N_0 = 13.2$ dB (or 20.89), and because each symbol is made up of ($\log_2 M$) bits, we compute, with $M = 8$:

$$\frac{E_s}{N_0} = (\log_2 M)\frac{E_b}{N_0} = 3 \times 20.89 = 62.67 \tag{17}$$

Using the results of Equation (17) in Equation (14) yields the symbol-error probability, $P_E = 2.2 \times 10^{-5}$. To transform this to bit-error probability, we need to use the relationship between bit-error probability P_B and symbol-error probability P_E, for multiple-phase signaling [9], as follows:

$$P_B \approx \frac{P_E}{\log_2 M} \qquad \text{for } P_E \ll 1 \tag{18}$$

which is a good approximation, when Gray coding [10] is used for the bit-to-symbol assignment. This last computation yields $P_B = 7.3 \times 10^{-6}$, which meets the required bit-error performance. Thus, in this example, no error-correction coding is necessary and 8-PSK modulation represents the design choice to meet the requirements of the bandwidth-limited channel (which we had predicted by examining the required E_b/N_0 values in Table 1).

Example 2: Power-Limited Uncoded System

Now, suppose that we have the same data rate and bit-error probability requirements as in the bandwidth-limited example. However, in this example, let the available bandwidth, W, be equal to 45 kHz, and let the available P_r/N_0 be equal to 48 dB-Hz. As before, the goal is to choose a modulation or modulation/coding scheme that yields the required performance. In this example, we will again find that error-correction coding is not required. The channel in this example is clearly not bandwidth-limited, since the available bandwidth of 45 kHz is more than adequate for supporting the required data rate of 9600 bit/s. The received E_b/N_0 is found from Equation (11), as follows:

$$\frac{E_b}{N_0} \text{ (dB)} = 48 \text{ dB-Hz} - (10 \times \log_{10} 9600) \text{ dB-bit/s} = 8.2 \text{ dB (or 6.61)} \tag{19}$$

Since there is abundant bandwidth but a relatively small amount of E_b/N_0 for the required bit-error probability, this channel may be referred to as *power-limited.* We therefore choose MFSK as the modulation scheme. In an effort to conserve power, we next search for the *largest possible M* such that the MFSK minimum bandwidth is not expanded beyond our available bandwidth of 45 kHz. From Table 1, we see that such a search results in the choice of $M = 16$. Our next task is to determine whether the required error performance of $P_B \leq 10^{-5}$ can be met by using 16-FSK alone, without the use of any error-correction coding. Similar to the previous example, Table 1 shows that 16-FSK *alone* will meet the requirements, since the required E_b/N_0 listed for 16-FSK is less than the received E_b/N_0 that was calculated in Equation (19). However, imagine again that we do not have Table 1. Let's see how to evaluate whether error-correction coding is necessary.

As before, the block diagram in Figure 2 summarizes the relationship between symbol rate R_s and bit rate R, and between E_S/N_0 and E_b/N_0, which is identical to each of the respective relationships in the previous bandwidth-limited example. In this example, the 16-FSK demodulator receives a waveform (one of 16 possible frequencies) during each symbol-time interval T_S. For noncoherent MFSK, the probability that the demodulator makes a symbol error, $P_E(M)$, is approximated by the following upper bound [11]:

$$P_E(M) \leq \frac{M-1}{2} \exp\left(-\frac{E_s}{2N_0}\right) \tag{20}$$

To solve for $P_E(M)$ in Equation (20), we need to compute E_S/N_0, as we did in Example 1. Using the results of Equation (19) in Equation (17), with $M = 16$, we get this:

$$\frac{E_s}{N_0} = (\log_2 M) \frac{E_b}{N_0} = 4 \times 6.61 = 26.44 \qquad (21)$$

Next, using the results of Equation (21) in Equation (20) yields the symbol-error probability $P_E = 1.4 \times 10^{-5}$. To transform this to bit-error probability, P_B, we need to use the relationship between P_B and P_E for orthogonal signaling [11], given by

$$P_B = \frac{2^{k-1}}{2^k - 1} P_E \qquad (22)$$

This last computation yields $P_B = 7.3 \times 10^{-6}$, which meets the required bit-error performance. Thus, we can meet the given specifications for this power-limited channel by using 16-FSK modulation, without any need for error-correction coding (which we had predicted by examining the required E_b/N_0 values in Table 1).

Example 3: Bandwidth-Limited and Power-Limited Coded System

In this example, we start with the same channel parameters as in the bandwidth-limited example, namely, $W = 4000$ Hz, $P_r/N_0 = 53$ dB-Hz, and $R = 9600$ bit/s, with one exception. In the present example, we specify that the bit-error probability must be *at most* 10^{-9}. Since the available bandwidth is 4000 Hz, and from Equation (12) the available E_b/N_0 is 13.2 dB, it should be clear from Table 1 that the system is both bandwidth-limited *and* power-limited. (8-PSK is the only possible choice to meet the bandwidth constraint; however, the available E_b/N_0 of 13.2 dB is certainly insufficient to meet the required bit-error probability of 10^{-9}.) For such a small value of P_B, the system shown in Figure 2 will obviously be inadequate, and we need to consider the performance improvement that error-correction coding (within the available bandwidth) can provide. In general, you can use convolutional codes or block codes. To simplify the explanation, we will choose a block code. The Bose, Chaudhuri, and Hocquenghem (BCH) codes form a large class of powerful error-correcting cyclic (block) codes [12]. For this example, let's select one of the codes from this family of codes. Table 2 presents a partial catalog of the available BCH codes in terms of *n, k,* and *t,* where *k* represents the number of information or data bits that the code transforms into a longer block of *n* code bits (also called *channel bits* or *channel symbols*), and *t* represents the largest number of incorrect channel bits that the code can correct within each *n*-sized block. The *rate* of a code is defined as the ratio *k/n*; its inverse represents a measure of the code's redundancy.

Table 2

BCH Codes (Partial Catalog)

n	*k*	*t*
7	4	1
15	11	1
	7	2
	5	3
31	26	1
	21	2
	16	3
	11	5
63	57	1
	51	2
	45	3
	39	4
	36	5
	30	6
127	120	1
	113	2
	106	3
	99	4
	92	5
	85	6
	78	7
	71	9
	64	10
	57	11
	50	13
	43	14
	36	15
	29	21
	22	23
	15	27
	8	31

Since this example is represented by the same bandwidth-limited parameters that were given in the first example, we start with the same 8-PSK modulation as before in order to meet the stated bandwidth constraint. However, we now also need to employ error-correction coding so that the bit-error probability can be lowered to $P_B \leq 10^{-9}$. To make the optimum code selection from Table 2, we are guided by the following goals:

1. The undetected bit-error probability of the combined modulation/coding system must meet the system error requirement.

2. The rate of the code must not expand the required transmission bandwidth beyond the available channel bandwidth.

3. The code should be as simple as possible. Generally, the shorter the code, the simpler its implementation.

The uncoded 8-PSK minimum bandwidth requirement is 3200 Hz (see Table 1) and the allowable channel bandwidth is specified as 4000 Hz. Therefore, the uncoded signal bandwidth may be increased by *no more than* a factor of 1.25 (or an expansion of 25%). Thus, the very first step in this (simplified) code-selection example is to eliminate the candidates from Table 2 that would expand the bandwidth by more than 25%. The remaining entries in Table 2 form a much reduced set of “bandwidth-compatible” codes, which have been listed in Table 3.

In Table 3, a column designated Coding Gain, G (for MPSK at $P_B = 10^{-9}$), has been added, where coding gain in decibels is defined as follows:

$$G\,(\text{dB}) = \left(\frac{E_b}{N_0}\right)_{\text{uncoded}} (\text{dB}) \; - \left(\frac{E_b}{N_0}\right)_{\text{coded}} (\text{dB}) \qquad (23)$$

From Equation (23), coding gain can be described as a measure of the *reduction* in the required E_b/N_0 (in decibels) that needs to be provided, due to the error-performance properties of the channel coding. Coding gain is a function of the particular code and modulation types used, and the bit-error probability. In Table 3, the coding gain, G, has been computed for MPSK at $P_B = 10^{-9}$. For MPSK modulation, G is relatively independent of the value of M. Thus, for a particular bit-error probability, a given code will provide approximately the same coding gain when used with any of the MPSK modulation schemes. The coding gains in Table 3 were calculated using a procedure outlined under the later section “Calculating Coding Gain.”

Table 3

Bandwidth-Compatible BCH Codes

n	*k*	*t*	Coding Gain, G (dB) MPSK, $P_B = 10^{-9}$
31	26	1	2.0
63	57	1	2.2
	51	2	3.1
127	120	1	2.2
	113	2	3.3
	106	3	3.9

Figure 3 shows a block diagram that summarizes the details of this system containing both modulation and coding. Compare Figure 3 with Figure 2; the introduction of the encoder/decoder blocks has brought about additional transformations. At the encoder/modulator, in Figure 3, the relationships are shown that exist when transforming from R bit/s to R_c channel-bit/s to R_S symbol/s.

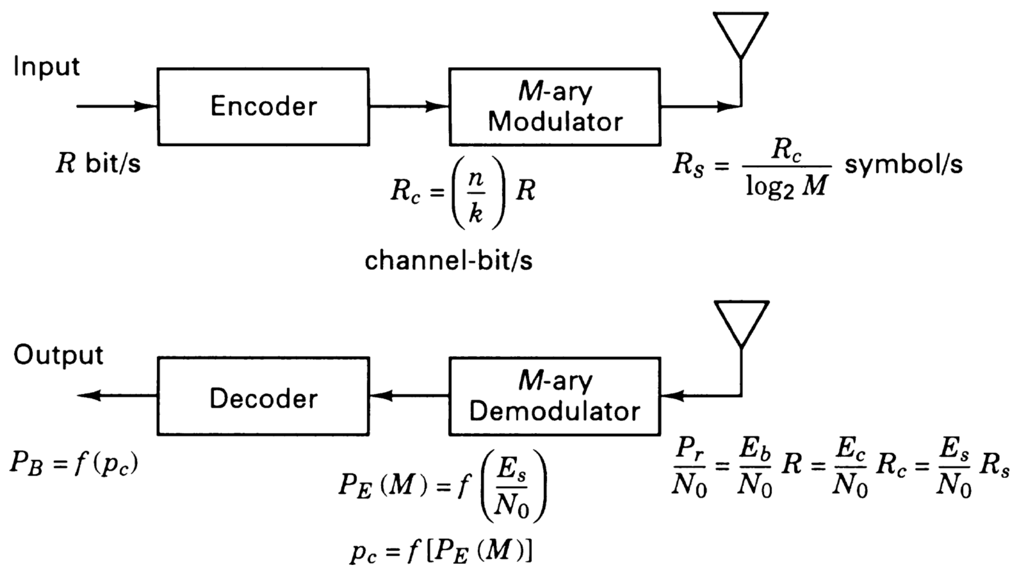

Figure 3

Modem with channel coding.

We assume that our communication system is a real-time system and thus cannot tolerate any message delay. Therefore, the channel-bit rate, R_c, must *exceed* the data-bit rate, R, by the factor n/k. Further, each transmission symbol is made up of

($\log_2 M$) channel bits, so the symbol rate, R_S, is *less* than R_c by the factor ($\log_2 M$). For a system containing both modulation and coding, we summarize the rate transformations as follows:

$$R_c = \left(\frac{n}{k}\right) R \tag{24}$$

$$R_s = \frac{R_c}{\log_2 M} \tag{25}$$

At the demodulator/decoder, in Figure 3, the transformations among data-bit energy, channel-bit energy, and symbol energy are related (in a reciprocal fashion) by the same factors as shown among the rate transformations in Equations (24) and (25). Since the encoding transformation has replaced k data bits with n channel bits, the ratio of channel-bit energy to noise-power spectral density, E_c/N_0, is computed by decrementing the value of E_b/N_0 by the factor k/n. Also, since each transmission symbol is made up of ($\log_2 M$) channel bits, E_S/N_0, which is needed in Equation (14) to solve for P_E, is computed by incrementing E_c/N_0 by the factor ($\log_2 M$). For a system containing both modulation and coding, we summarize the energy to noise-power spectral density transformations as follows:

$$\frac{E_c}{N_0} = \left(\frac{k}{n}\right) \frac{E_b}{N_0} \tag{26}$$

$$\frac{E_s}{N_0} = (\log_2 M) \frac{E_c}{N_0} \tag{27}$$

Therefore, using Equations (24) through (27), we can now expand the expression for P_r/N_0 in Equation (13) as follows:

$$\frac{P_r}{N_0} = \frac{E_b}{N_0} R = \frac{E_c}{N_0} R_c = \frac{E_s}{N_0} R_s \tag{28}$$

As before, a standard way of describing the link is in terms of the received E_b/N_0 in decibels. However, *there are no data bits* at the input to the demodulator/detector; *neither are there any channel bits.* There are *only* waveforms (transmission symbols) that have bit meanings, and thus the waveforms can be described in terms of bit-energy apportionments. Equation (28) illustrates that the predetection point in the receiver is a useful reference point at which we can relate the *effective* energy and the *effective* speed of various parameters of interest. We use the word "effective" because the only type of signals that actually appear at the predetection point are waveforms (transformed to baseband pulses) that we call symbols. Of course, these symbols are related to channel bits, which in turn are related to data bits. To emphasize the point that Equation (28) represents a useful kind of "bookkeeping," consider a system in which a stream of some number of bits, say 273 bits, appears so repeatedly as a module that we give this group of 273 bits a name; we call it a "chunk." Engineers do that all the time—for example, eight bits are referred to as a *byte*. The moment we identify this new entity, the chunk, it can immediately be related to the parameters in Equation (28), since P_r/N_0 will now also equal the energy in a chunk over N_0, times the chunk rate.

Since P_r/N_0 and R were given as 53 dB-Hz and 9600 bit/s, respectively, we find as before from Equation (12) that the received E_b/N_0 = 13.2 dB. Note that the received E_b/N_0 is fixed and independent of the code parameters n and k, and the modulation parameter M. As we search in Table 3 for the ideal code that will meet the specifications, we can iteratively repeat the computations that are summarized in Figure 3. It might be useful to program on a PC (or calculator) the following four steps as a function of n, k, and t. Step 1 starts by combining Equations (26) and (27), as follows:

Step 1:
$$\frac{E_s}{N_0} = (\log_2 M)\ \frac{E_c}{N_0} = (\log_2 M)\left(\frac{k}{n}\right)\frac{E_b}{N_0} \tag{29}$$

Step 2:
$$P_E(M) \approx 2Q\left[\sqrt{\frac{2E_s}{N_0}}\ \sin\left(\frac{\pi}{M}\right)\right] \tag{30}$$

The expression in step 2 is the approximation (for M-ary PSK) for symbol-error probability, P_E, rewritten from Equation (14). At each symbol-time interval, the demodulator makes a symbol decision, but it delivers to the decoder a channel-bit sequence representing that symbol. When the channel-bit output of the demodulator is quantized to two levels, denoted by 1 and 0, the demodulator is said to make *hard decisions*. When the output is quantized to more than two levels, the demodulator is said to make *soft decisions*. Throughout this section, hard-decision demodulation is assumed.

Now that a decoder block is present in the system, we designate the channel-bit-error probability out of the demodulator and into the decoder as P_c, and reserve the notation P_B for the bit-error probability *out of the decoder* (the decoded bit-error probability). Equation (18) is rewritten in terms of P_c as follows:

Step 3: $$p_c \approx \frac{P_E}{\log_2 M} \qquad \text{for } P_E \ll 1 \tag{31}$$

Relating the channel-bit-error probability to the symbol-error probability out of the demodulator, assuming Gray coding, as referenced in Equation (18).

For a real-time communication system, using traditional channel-coding schemes, and a given value of received P_r/N_0, the value of E_S/N_0 with coding will *always be less* than the value of E_S/N_0 without coding. Since the demodulator, with coding, receives less E_S/N_0, it makes more errors! However, when coding is used, the system error-performance doesn't depend only on the performance of the demodulator; it also depends on the performance of the decoder. Thus, for error-performance improvement due to coding, we require that the decoder provide enough error correction to *more than compensate* for the poor performance of the demodulator. The final output decoded bit-error probability, P_B, depends on the particular code, the decoder, and the channel-bit-error probability, P_c. It can be expressed [13] by the following approximation:

Step 4: $$P_B \approx \frac{1}{n} \sum_{j=t+1}^{n} j \binom{n}{j} p_c^{\,j} (1 - p_c)^{n-j} \tag{32}$$

where t is the largest number of channel bits that the code can correct within each block of n bits. Using Equations (29) through (32) in the above four steps, the decoded bit-error probability, P_B, can be computed as a function of n, k, and t for each of the codes listed in Table 3. The entry that meets the stated error requirement with the *largest possible* code rate and the *smallest* value of n is the double-error-correcting (63, 51) code. The computations are as follows:

Step 1: $$\frac{E_s}{N_0} = 3 \left(\frac{51}{63}\right) 20.89 = 50.73$$

where $M = 8$, and the received $E_b/N_0 = 13.2$ dB (or 20.89).

Step 2: $$P_E \approx 2Q\left[\sqrt{101.5} \times \sin\left(\frac{\pi}{8}\right)\right] = 2Q\,(3.86) = 1.2 \times 10^{-4}$$

Step 3: $$p_c \approx \frac{1.2 \times 10^{-4}}{3} = 4 \times 10^{-5}$$

Step 4: $$P_B \approx \frac{3}{63}\binom{63}{3}(4 \times 10^{-5})^3 \ (1 - 4 \times 10^{-5})^{60}$$

$$+ \frac{4}{63}\binom{63}{4}(4 \times 10^{-5})^4 \ (1 - 4 \times 10^{-5})^{59} + \ldots$$

$$= 1.2 \times 10^{-10}$$

where the bit-error-correcting capability of the code is $t = 2$. For the computation of P_B in Step 4, only the first two terms in the summation of Equation (32) have been used, since the other terms have a vanishingly small effect on the result whenever P_c is small or E_b/N_0 reasonably large. When performing this computation with a computer, it is important to *always* include all of the summation terms in Equation (32), since a truncated solution can be very erroneous whenever E_b/N_0 is small. Now that we have selected the (63, 51) code, the values of channel-bit rate, R_c, and symbol rate, R_S, are computed using Equations (24) and (25), with $M = 8$.

$$R_c = \left(\frac{n}{k}\right) R = \left(\frac{63}{51}\right) 9600 \approx 11{,}859 \text{ channel-bits/s}$$

$$R_s = \frac{R_c}{\log_2 M} = \frac{11859}{3} = 3953 \text{ symbols/s}$$

Calculating Coding Gain

Perhaps a *more direct way* of finding the simplest code that meets the specified error performance is to first compute how much coding gain, G, would be required in order to yield $P_B = 10^{-9}$ when using 8-PSK modulation alone; and then we can simply choose from Table 3 the code that provides this performance improvement. First, the *uncoded* E_S/N_0 that will yield an undetected error probability of $P_B = 10^{-9}$ is found by writing from Equations (18) and (30):

$$P_B \approx \frac{P_E}{\log_2 M} \approx \frac{2Q\left[\sqrt{\frac{2E_s}{N_0}}\,\sin\left(\frac{\pi}{M}\right)\right]}{\log_2\ M} = 10^{-9} \tag{33}$$

At this low value of bit-error probability, it is valid to use Equation (16) to approximate $Q(x)$ in Equation (33). By trial and error (on a programmable calculator), we find that the *uncoded* E_S/N_0 = 120.67 = 20.8 dB, and since each symbol is made up of ($\log_2$ 8) = 3 bits, the required $(E_b/N_0)_{\text{uncoded}}$ = 120.67/3 = 40.22 = 16 dB. We know from the given parameters in this example and Equation (12) that the received $(E_b/N_0)_{\text{coded}}$ = 13.2 dB. Therefore, using Equation (23), the required coding gain to meet the bit-error performance of $P_B = 10^{-9}$ is

$$G\,(\text{dB}) = \left(\frac{E_b}{N_0}\right)_{\text{uncoded}} (\text{dB}) - \left(\frac{E_b}{N_0}\right)_{\text{coded}} (\text{dB}) = 16\ \text{dB} - 13.2\ \text{dB} = 2.8\ \text{dB}$$

To be precise, each of the E_b/N_0 values in the above computation must correspond to the same value of bit-error probability (which they do not). They correspond to $P_B = 10^{-9}$ and $P_B = 1.2 \times 10^{-10}$, respectively. However, at these low probability values, even with such a discrepancy this computation still provides a good approximation of the required coding gain. Searching Table 3 for the simplest code that will yield a coding gain of *at least* 2.8 dB, we see that the choice is the (63, 51) code, which corresponds to the same code choice that was made earlier. Note that coding gain must always be specified for a particular error probability and modulation type, as it is in Table 3.

Example 4: Direct-Sequence (DS) Spread-Spectrum Coded System

Spread-spectrum systems are not usually classified as being bandwidth- or power-limited. However, they are generally perceived to be power-limited systems because the bandwidth occupancy of the information is much larger than the bandwidth that is intrinsically needed for the information transmission. In a direct-sequence spread-spectrum (DS/SS) system, spreading the signal bandwidth by some factor permits lowering the signal-power spectral density by the same factor (the total average signal power is the same as before spreading). The bandwidth spreading is typically accomplished by multiplying a relatively narrowband data signal by a wideband spreading signal. The spreading signal or *spreading code* is often referred to as a *pseudorandom code* or PN code.

A typical DS/SS radio system is often described as a two-step BPSK modulation process. The first step can be viewed as the modulation of a carrier wave by a bipolar data waveform having a value +1 or –1 during each data-bit duration. In the second step, the output of the first step is multiplied (modulated) by a bipolar PN-code waveform having a value +1 or –1 during each PN-code-bit duration. In reality, DS/SS systems are usually implemented by first multiplying the data waveform by the PN-code waveform, and then making a single pass through a BPSK modulator. However, for this example, it will be useful to characterize the modulation process in two separate steps—the outer modulator/demodulator for the data, and the inner modulator/demodulator for the PN code.

A spread-spectrum system is characterized by a *processing gain*, G_p, that is defined in terms of the spread-spectrum bandwidth, W_{ss}, and the data rate, R, as follows [4]:

$$G_p = \frac{W_{ss}}{R} \tag{34}$$

For a DS/SS system, the PN-code bit has been given the name *chip*, and the spread-spectrum signal bandwidth can be shown to be approximately equal to the chip rate. Thus, for a DS/SS system, the processing gain in Equation (34) is generally expressed in terms of the chip rate, R_{ch}, as follows:

$$G_p = \frac{R_{ch}}{R} \tag{35}$$

It is worth noting that some authors define *processing gain* to be the ratio of the spread-spectrum bandwidth to the symbol rate. This definition separates the system performance due to bandwidth spreading from the performance due to error-correction coding. Since we ultimately want to relate all of the coding mechanisms relative to the information source, we will conform to the definition for processing gain, as expressed in Equations (34) and (35).

A spread-spectrum system can be used for interference rejection and for multiple access (allowing multiple users to access a communications resource simultaneously). The benefits of DS/SS signals are best achieved when the processing gain is very large; in other words, the chip rate of the spreading (or PN) code is much larger than the data rate. In such systems, the large value of G_p allows the signaling chips to be transmitted at a power level well below that of the thermal noise. At the receiver, the despreading operation correlates the incoming signal with a synchronized copy of the PN code, and thus accumulates the energy from multiple (G_p) chips to yield the energy per data bit. The value of G_p has a major influence on the performance of the spread-spectrum system application. However, we will see that the value of G_p has *no effect* on the value of the received E_b/N_0. In other words, spread-spectrum techniques offer no error-performance advantage over thermal noise. For DS/SS systems, there is no disadvantage, either. Sometimes such spread-spectrum radio systems are employed *only* to enable the transmission of very small power-spectral densities, and thus avoid the need for FCC licensing [14].

For this example, consider a DS/SS radio system that uses the same (63, 51) code as in Example 3. However, instead of using MPSK for the data modulation, we will use BPSK, and we will use BPSK for modulating the PN-code chips. Let the received $P_r/N_0 =$ 48 dB-Hz, the data rate, $R = 9600$ bit/s, and the required $P_B \leq 10^{-6}$. For simplicity, assume that there are no bandwidth constraints. In this example, our task is simply to determine whether the required error performance can be achieved using the given system architecture and design parameters. In evaluating the system, we will use the same type of transformations that were used in the previous examples.

When we consider the relationships in transforming from data bits to channel bits to symbols to chips, we can see the same pattern of subtle but straightforward transformations in rates and energies as in Figures 2 and 3. The values of R_c, R_S, and R_{ch} can now be calculated immediately, since the (63, 51) BCH code has already been selected. From Equation (24):

$$R_c = \left(\frac{n}{k}\right) R = \left(\frac{63}{51}\right) 9600 \approx 11{,}859 \text{ channel-bit/s}$$

Since the data modulation considered here is BPSK,

$$R_S = R_c \approx 11{,}859 \text{ symbol/s}$$

and, from Equation (35), with an assumed value of $G_P = 1000$

$$R_{ch} = G_P\, R = 1000 \times 9600 = 9.6 \times 10^6 \text{ chip/s}$$

Since, in this example, we have been given the same P_r/N_0 and the same data rate as in Example #2, we find the value of received E_b/N_0 from Equation (19) to be 8.2 dB (or 6.61). At the demodulator, we can now expand the expression for P_r/N_0 in Equation (28) as follows:

$$\frac{P_r}{N_0} = \frac{E_b}{N_0} R = \frac{E_c}{N_0} R_c = \frac{E_s}{N_0} R_s = \frac{E_{\text{ch}}}{N_0} R_{\text{ch}} \tag{36}$$

Corresponding to each transformed entity (data bit, channel bit, symbol, or chip) there is a change in rate, and similarly, a reciprocal change in energy-to-noise spectral density for that received entity. It should be apparent that Equation (36) is valid for any such transformation when the rate and energy are modified in a reciprocal way. There is a kind of *conservation of power* (or *energy*) phenomenon that exists in the transformations. The total received average power (or total received energy per symbol duration) is *fixed* regardless of how it is computed—on the basis of data bits, channel bits, symbols, or chips. Thus, even though the ratio E_{ch}/N_0 is much less in value than E_b/N_0, which can be seen from Equations (36) and (35) as follows:

$$\frac{E_{\text{ch}}}{N_0} = \frac{P_r}{N_0}\left(\frac{1}{R_{ch}}\right) = \frac{P_r}{N_0}\left(\frac{1}{G_p\ R}\right) = \left(\frac{1}{G_p}\right)\frac{E_b}{N_0} \tag{37}$$

The despreading function (when properly synchronized) will accumulate the energy contained in a quantity G_p of the chips, yielding the same value of $E_b/N_0 = 8.2$ dB, as was computed earlier from Equation (19). Therefore, the DS spreading transformation has no effect on the error performance of an AWGN channel [4], and thus the value of G_p will have no bearing on the resulting value of P_B in this example. From Equation (37), we can compute

$$\frac{E_{\text{ch}}}{N_0}\ (\text{dB}) = \frac{E_b}{N_0}\ (\text{dB}) - G_P\ (\text{dB}) \tag{38}$$

$$= 8.2\ \text{dB} - (10 \times \log_{10} 1000)\ \text{dB}$$

$$= -21.8\ \text{dB}$$

It is interesting to note that the value of processing gain in this example ($G_p = 1000$) enables the DS/SS system to operate at a value of chip energy *well below the thermal noise,* with the same error performance as without spreading. Since BPSK is the data modulation selected here, each message symbol therefore corresponds to a single channel bit, and we can write the following:

$$\frac{E_s}{N_0} = \frac{E_c}{N_0} = \left(\frac{k}{n}\right)\frac{E_b}{N_0} = \left(\frac{51}{63}\right) \times 6.61 = 5.35 \tag{39}$$

where the received E_b/N_0 = 8.2 dB (or 6.61). Out of the BPSK data demodulator, the symbol-error probability, P_B, (and the channel-bit error probability, p_c) is computed as [4]:

$$p_c = P_E = Q\left(\sqrt{\frac{2\ E_c}{N_0}}\right) \tag{40}$$

Using the results of Equation (39) in Equation (40) yields the following:

$$Pc = Q(3.27) = 5.8 \times 10^{-4}$$

Finally, using this value of p_c in Equation (32) for the (63, 51) double-error correcting code yields the output bit-error probability of $P_B = 3.6 \times 10^{-7}$. We can therefore verify that for the given architecture and design parameters of this example, the system does in fact achieve the required error performance.

Code Selection

Consider a real-time communication system, in which the specifications cause it to be power-limited but there is ample available bandwidth, and the users require a very small bit-error probability. Error-correction coding is called for. Suppose that we were asked to select one of the BCH codes listed in Table 2. Since the system is not bandwidth-limited, and it requires very good error performance, one might be tempted to simply choose the most powerful code in Table 2, that is, the (127, 8) code, capable of correcting any combination of up to 31 flawed bits within a block of 127 code bits. Would anyone use such a code in a real-time communication system? No, they wouldn't. Let me explain why such a choice would be *unwise*.

Whenever error-correction coding is used in a real-time communication system, there are two mechanisms at work that influence error performance. One mechanism works to improve the performance, and the other works to degrade it. The improving-mechanism is the coding; the greater the redundancy, the greater will be the error-correcting capability of the code. The degrading mechanism is the energy reduction per channel symbol or code bit (compared to the data bit). This reduced energy stems from the increased redundancy (and faster signaling in a real-time communication system). The reduced symbol energy causes the demodulator to make more errors. Eventually, the second mechanism wins out, and thus at very low code rates we see degradation. This is demonstrated in Example 5 below. Note that the degrading mechanism applies for coding only in a real-time system (where messages cannot be delayed). For systems that can endure message delays, the tradeoff for getting the benefits of the code redundancy is delay (not reduced symbol energy).

Example 5: Choosing a Code to Meet Performance Requirements

A system is specified with the following parameters: P_r/N_0 = 67 dB-Hz, data rate $R = 10^6$ bits/s, available bandwidth W = 20 MHz, decoded bit-error probability $P_B \leq 10^{-7}$, and the modulation is BPSK. Choose a code from Table 2 that will fulfill these requirements. Start by considering the (127, 8) code. It appears attractive because it has the greatest bit-error correcting capability on the list.

The (127, 8) code expands the transmission bandwidth by a factor of 127/8 = 15.875. Hence, the signaling rate of 1 Mbit/s (giving rise to a nominal bandwidth of 1 MHz) will be expanded to 15.875 MHz by using this code. The transmission signal is within the available bandwidth of 20 MHz, even after allowing another 25% bandwidth expansion for filtering. After choosing this code, we next evaluate the error performance, by following the steps outlined earlier, which yields the following:

$$\frac{E_b}{N_0} = \frac{P_r}{N_0}\left(\frac{1}{R}\right) = 67\text{ dB} - 60\text{ dB} = 7\text{ dB} \ (\text{or } 5)$$

$$\frac{E_s}{N_0} = \frac{E_c}{N_0} = \left(\frac{k}{n}\right)\frac{E_b}{N_0} = \left(\frac{8}{127}\right)5 = 0.314$$

Since the modulation is binary, $p_c = P_E$, so that

$$p_c = P_E \approx Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = Q\left(\sqrt{0.628}\right) = Q\left(0.7936\right) = 0.2156$$

Since the (127, 8) code is a t = 31 error-correcting code, we next use Equation (32) to find the decoded bit-error probability, as follows:

$$P_B \approx \frac{1}{n}\sum_{j=t+1}^{n} j\binom{n}{j} p_c^j (1-p_c)^{n-j} = \frac{1}{127}\sum_{j=32}^{127} j\binom{127}{j}(0.2156)^j \ (1-0.2156)^{127-j}$$

Whenever p_c is very small, it suffices to use only the first term, or the first few terms in the summation. But when p_c is large, as here, computer assistance is helpful. Solving the above with $p_c = 0.2156$ yields a decoded bit-error probability of $P_B = 0.05$, which is a far cry from the system requirement of 10^{-7}. Next, let's select a code whose code rate is close to the popular rate ½—that is the (127, 64) code. It is not as capable as the first choice because it corrects only 10 flawed bits in a block of 127 code bits. But watch what happens. Using the same steps as before yields this:

$$\frac{E_s}{N_0} = \frac{E_c}{N_0} = \left(\frac{k}{n}\right)\frac{E_b}{N_0} = \left(\frac{64}{127}\right)5 = 2.519$$

Notice how much larger the E_S/N_0 is here, compared to the case where the (127, 8) code was used. This larger E_S/N_0 results in smaller values of p_c and P_B, as seen by

$$p_c = Q\left(\sqrt{2\times 2.519}\right) = Q\left(\sqrt{2.245}\right) = 0.0124$$

$$P_B \approx \frac{1}{127}\sum_{j=11}^{127} j\binom{127}{j}(0.0124)^j\ (1-0.0124)^{127-j}$$

The result yields $P_B = 5.6 \times 10^{-8}$, which meets the system requirements. From this example, you should see that the selection of a code needs to be made in concert with the modulation choice and the available E_b/N_0. Be guided by the fact that very high rates and very low rates generally perform poorly in a real-time communication system. As was described earlier, this comes about because there are two mechanisms at work: (1) an improving mechanism; more redundancy means greater error-correcting capability, and (2) a degrading mechanism; energy reduction per channel symbol causes the demodulator to make more errors. As the code rate is reduced, the second mechanism eventually wins out, and thus at very low code rates the system experiences error-performance degradation [4].

Conclusion

The goal in this article has been to review fundamental relationships used in designing digital communication systems. First, we examined the concept of bandwidth-limited and power-limited systems and how such conditions influence the design. Most importantly, we focused on the definitions and computations involved in transforming from data bits to channel bits to symbols to chips. In general, most digital communication systems share these relationships; thus, understanding them should enable you to apply the same concepts to other such systems.

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